Math 131B-2: Homework 5

Due: May 5, 2014

1. Read Apostol Sections 4.16-17, 4.19-20, 9.1-5.
2. Do examples (a)-(d) of problem 4.11 in Apostol. [You do not have to do the proof at the beginning.]
3. Do problems 4.28, 4.37, 4.49, 4.52, 4.54 in Apostol.

   [Note that it is very important that 4.52 involves a subset of $\mathbb{R}^n$; this fact is *not* true in general.]
4. Decide whether the following sequences converge pointwise on the domain specified. If so, to what?
   - $f_n(x) = nx(1-x)^n$ on $[0, 1]$
   - $f_n(x) = \frac{\sin(nx+3)}{\sqrt{n+1}}$ on $[0, 1]$.
   - $f_n(x) = \cos^n(x)$ on $[0, 2\pi]$.
   - $f_n(x) = n^2 x^n$ on $[0, 1]$.
5. Let $f_n(x) = x^n$ on $[0, 1]$.
   - Show that the sequence $\{f_n\}$ converges pointwise, but not uniformly on $[0, 1]$.
   - Let $g : [0, 1] \to \mathbb{R}$ be any continuous function on $[0, 1]$ such that $g(1) = 0$. Show that the sequence $\{g \cdot f_n\}$ converges uniformly. [Hint: Break the interval into two pieces.]
6. We say a sequence of real-valued functions is *uniformly bounded* if there is a single $M > 0$ such that $|f_n(x)| < M$ for all n and all x in the domain. Prove that any uniformly convergent sequence of bounded real-valued functions $\{f_n\}$ is uniformly bounded.